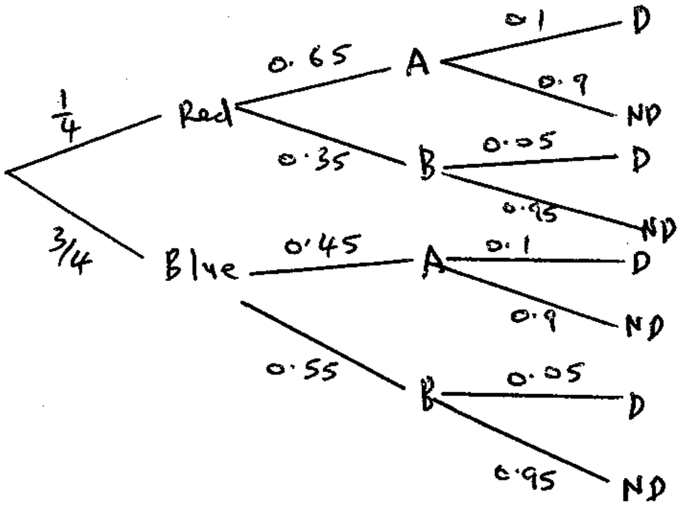
January 2006
6687 Statistics S5
Mark Scheme

Question Number	Scheme	Marks
1 (a)	Geometric	B1 (1)
(b)(i)	$p = \frac{1}{5}$	B1
	$\therefore P(J=4) = \left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right) = \frac{64}{625} = 0.1024$	M1 A1 (3)
(ii)	$P(J \geqslant 3) = (1-p)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25} = 0.64$	M1 A1 √A1 (3)
(c)	Assume shots are independent and	B1
	probability of a hit is constant.	B1 (2)
		TOTAL 9
2		
(a)	$P(D) = \left(\frac{1}{4}\times 0.65\times 0.10\right) + \left(\frac{1}{4}\times 0.35\times 0.05\right)$ 4 cases	M1
	$+ \left(\frac{3}{4}\times 0.45\times 0.1\right) + \left(\frac{3}{4}\times 0.55\times 0.05\right)$ 2 terms $\frac{1}{4}$	A1
	2 terms $3/4$	A1
	$= 0.075$ A1	(4)
(b)	$P(A\mid D) = \dfrac{\left(\frac{1}{4}\times 0.65\times 0.1\right) + \left(\frac{3}{4}\times 0.45\times 0.1\right)}{0.075}$ Bayes	M1 $\frac{\text{M1 A1}}{\text{A1√}}$
	$= 2/3$ or $0.\dot{6}$	A1 (5)
		TOTAL 9

Question Number	Scheme	Marks
3 (a)	Uniform | Rectangular	B1 (1)
(b)	$M_X(t) = \int_0^4 0.25 e^{tx}\,dx$ Use of $M_X(t)$	M1A1
	$= \left[\dfrac{0.25e^{tx}}{t} \right]_0^4$	A1
	$= \dfrac{0.25e^{4t}}{t} - \dfrac{0.25}{t}$	
	$* = \dfrac{1}{4t}(e^{4t}-1)$ * AG cso	A1 (4)
(c)	$M_X(t) = \dfrac{4t + \frac{(4t)^2}{2!} + \frac{(4t)^3}{3!} + \ldots}{4t}$	M1
	$= 1 + 2t + \dfrac{8t^2}{3} + \ldots$	A1
	$\mu = M'(0) = 2$; $\sigma^2 = M''(0) - (M'(0))^2 = \dfrac{4}{3}$	A1 A1 (4)
		TOTAL 9
4(a)(i)	$\binom{6}{1}(0.15)(0.85)^5(0.15) = 0.059900\ldots$	M1A1 (2)
(ii)	$\dfrac{3}{0.15} = 20$	B1 (1)
(b)	Probability that John wins a coconut in a game is constant Games are independent	B1 B1 (2)
(c)	$\dfrac{r}{p} = 18$; $\dfrac{r(1-p)}{p^2} = 36$	B1 B1
	$\therefore\ 18(1-p) = 36p$	M1
	$p = \dfrac{1}{3} > 0.15 \Rightarrow$ Sue	A1 A1ft (5)
		TOTAL 10

5(a)	Let X represent the no. of power cuts per month.	
	$X \sim Po(0.25)$	M1
	$P(X=1) = 0.25e^{-0.25} = 0.1947$	A1 (2)
(b)	$P(0 \leq T \leq t) = \int_0^t 0.25e^{-0.25x}\,dx = \left[-e^{-0.25x}\right]_0^t$	M1 [A1]
	$* = 1 - e^{-t/4} *$ AG cso	A1 (3)
	[OR	
	$P(0 \leq T \leq t) = P(\text{At least one cut in } (0,t))$	B1
	$= 1 - P(\text{No cuts in } (0,t))$	M1
	$= 1 - e^{-t/4}$	A1 (3)]
(c)(i)	$P(4 \text{ months or less}) = 1 - e^{-4/4} = 0.63212$	M1 A1
(ii)	$P(\text{Between 3 and 4 months}) = 0.63212 - (1 - e^{-3/4})$	M1
	$= 0.10447\ldots$	A1 (4)
(d)	$P(\text{No cuts}) = 0.75$	M1
	$e^{-t/4} = 0.75$	M1
	$t = 1.150728\ldots$	A1 (3)
		TOTAL 12
6 (a)	$P(\text{Batch accepted after 1st sample}) = (1-0.05)^{10} = 0.598736\ldots$	B1 (1)
(b)	$P(9 \text{ pass}) + P(8 \text{ pass}) = 10(0.95)^9(0.05) + 45(0.95)^8(0.05)^2$	M1 A1 A1
	$= 0.389759\ldots$	A1 (4)
(c)	$P(\text{Accepted after 2nd sample}) = 10(0.95)^9(0.05)(0.95)^{10} + \left[10(0.95)^9(0.05)\right]^2$	M1 A1 A1
	$+ 45(0.95)^8(0.05)^2(0.95)^{10}$	
	$= 0.1866768\ldots + 0.09930357\ldots + 0.0446866\ldots$	A1
	$= 0.332666991\ldots$	A1
	$P(\text{Acceptance}) = 0.598736\ldots + 0.332666991\ldots$	M1
	$= 0.93140393$	A1 (7)
		TOTAL 12

7(a)	$G_X(1) = 1$	M1
	$k(1+2+2)^2 = 1 \Rightarrow k = \frac{1}{25}$ * AG cso	A1 (2)
(b)	$\frac{1}{25}(1+2t+2t^2)(1+2t+2t^2) = \frac{1}{25}(1+4t+8t^2+\ldots)$	M1
	$P(X=2) = \frac{8}{25}$	A1 (2)
(c)	$G'_X(t) = \frac{2}{25}(1+2t+2t^2)(2+4t)$	M1A1
	$G'_X(1) = \frac{12}{5} \Rightarrow E(X) = \underline{\frac{12}{5}}$	A1
	$G''_X(t) = \frac{2}{25}(2+4t)(2+4t) + \frac{2}{25}(1+2t+2t^2).4$	M1A1
	$G''_X(1) = \frac{112}{25}$	A1
	$Var(X) = \frac{112}{25} + \frac{12}{5} - \left(\frac{12}{5}\right)^2 = \underline{\frac{28}{25}}$	M1A1 (8)
(d)	$\frac{t}{25}(1+2t^2+2t^4)^2$	B1B1 (2)
		TOTAL 14